A Model of Muscle Geometry for a Two Degree-Of-Freedom Planar Arm

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Abstract

A muscle's action on a joint not only depends upon the muscle's size and activation level, but also upon the mechanical advantage (or moment arm) of the muscle upon the joint. This relationship is made more complex by the fact that the mechanical advantage can change drastically with skeletal configuration. Here, we describe a model of muscle geometry for several muscles involved in elbow and shoulder actuation. The model captures the gross changes in muscle moment arm while preserving reasonable computational efficiency which facilitates its use in simulation.

1 Introduction

In constructing a model of muscle action on a limb, we ultimately wish to compute the torques that are applied to the joints as a function of the current state of the limb (as described by joint position and velocity) and set of descending motor commands. In Fagg et al. (2001a) we develop a model of muscle force production with the stretch reflex intact. We then apply this model to the generation of elbow movements (Fagg et al., 2001b). The skeleto-muscular geometry is used in two distinct stages of this latter work. First, in order to make use of the muscle model, it is necessary to know the instantaneous change in muscle length. This quantity can be computed by knowing how a muscle changes its length as a function of changes in joint angles (i.e. $\frac{\partial L_i}{\partial \theta_j}$). Second, once forces are computed by the muscle model, the application of torques about the various joints requires knowledge of the muscle

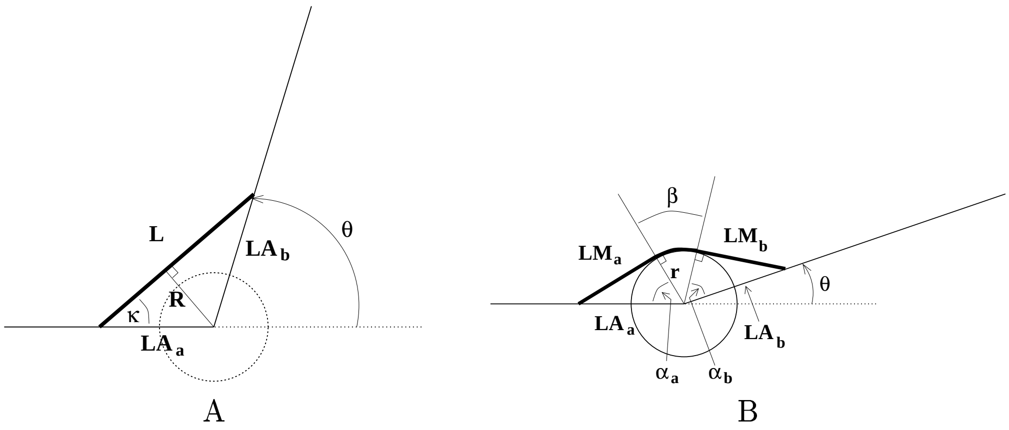

Figure 1: A simple muscle path model: straight path (A) and wrapped path (B). The critical parameters are: *theta* (joint orientation), LA_a and LA_b (distance from the center of joint rotation to muscle origin and insertion, respectively), r (the radius of the joint capsule), R (the muscle moment arm).

moment arms for each joint. For some muscles, this moment arm is constant; in others, the moment arm varies as a function of skeletal configuration.

In order to compute both of these quantities, it is necessary to take into account the attachment points of the muscle to the skeletal structure, as well as the path that the muscle takes between these points. Winters and Stark (1988) suggested a simple model of path in which the muscle is assumed to follow a straight line between muscle origin and insertion, except for highly extended positions. In the latter case, the muscle is assumed to wrap around a spherical joint capsule whose center is also the joint's center of rotation (figure 1).

The work of Amis, Dowson, and Wright (1979) and others (An et al., 1981) indicates that this form of path model captures the primary variation of muscle moment arms as a function of joint orientation for a number of elbow muscles, including the biceps long head and the triceps lateral head. In the latter case, however, the muscle wraps around the joint capsule for all feasible joint configurations, and hence the moment arm is assumed to be constant. Less is known about the geometry of muscles involving the shoulder, we assume for simplicity that this path model also applies in this case.

In our model, we follow Gribble et al (1997, 1998) in making use of six *equivalent muscles* to drive movements of the shoulder and elbow. The primary muscle groups contributing to movements are assumed to be: the pectoralis (mono-articulate shoulder flexors), deltoid (shoulder extensors), biceps long head (mono-articulate elbow flexors), triceps lateral head (elbow extensors), biceps short head (bi-articulate flexors), and triceps long head (bi-articulate extensors). All three extensors are assumed to wrap around the associated joint capsule at all times. The mono-articulate shoulder and elbow flexors are assumed to follow the above path model, and the biarticulate flexor follows a two-joint generalization of this model (as has been done by Gribble 1998).

Note that for a given flexor muscle and joint, it can be shown for both the wrap and no-wrap conditions that $\frac{\partial L}{\partial \theta} = -R$, where R is the muscle moment arm about the joint.[1] This relationship changes sign for extensors. This leaves us with the task of computing the muscle lengths and moment arms for both the mono-articulate and bi-articulate flexor muscles, which we will do in the following sections.

2 Single-Joint Case

The two conditions for the single-joint case are illustrated in figure 1. We first determine the joint boundary between these two cases, and then compute the necessary quantities for each condition.

2.1 Wrap/No Wrap Boundary

The boundary between the wrap and no wrap conditions occurs where the muscle contacts the joint capsule at a single point, as illustrated in figure 2. The critical quantities are as follows:

$$\begin{aligned}
\alpha_a &= \arccos\left(\frac{r}{LA_a}\right), && (1)\\
\alpha_b &= \arccos\left(\frac{r}{LA_b}\right), && (2)\\
\theta^c &= \pi - \alpha_a - \alpha_b, && (3)\\
LM_a &= LA_a \sin(\alpha_a), && (4)\\
LM_b &= LA_b \sin(\alpha_b), && (5)\\
L &= LM_a + LM_b,
\end{aligned}$$

where r is the radius of the joint capsule, and LA_a and LA_b are the distances from the muscle attachment points to the center of joint rotation. θ^c is the joint angle at which the transition between the two conditions occurs. α_a, α_b, LM_a, and LM_b are constant properties of the triangles formed by the center of rotation, the muscle attachment point, and the first point at which the muscle contacts the joint capsule, and will be used for the wrap condition.

2.2 No Wrap Condition

The no-wrap condition applies if $\theta \geq \theta^c$. The necessary quantities are computed as follows (see Figure 1A):

[1] We define an elbow joint angle of zero as full extension and flexion as a positive joint movement.

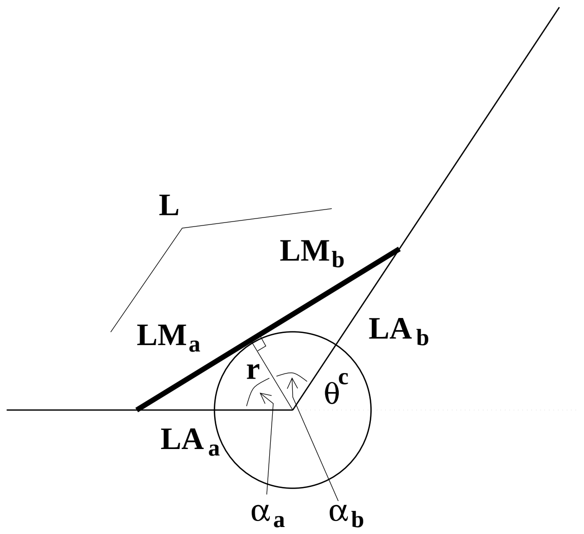

Figure 2: Wrap boundary.

$$
\begin{aligned}
L &= \sqrt{LA_a^2 + LA_b^2 + 2LA_a LA_b \cos(\theta)}, \\
\kappa &= \arccos\left(\frac{LA_a + LA_b \cos(\theta)}{L}\right), \\
R &= LA_a \sin(\kappa),
\end{aligned}
$$

where R is the moment arm for the muscle about the joint.

2.3 Wrap Condition

The wrap condition is shown in Figure 1B. The relevant quantities are:

$$
\begin{aligned}
\beta &= \pi - \theta - \alpha_a - \alpha_b, \\
L &= LM_a + r\beta + LM_b, \\
R &= r,
\end{aligned}
$$

where α_a, α_b, LM_a, and LM_b are computed in equations 1-5.

2.4 Monoarticular Example

For the biceps long head, Amis et al. (1979) report that the distance from biceps insertion to center of elbow rotation (our LM_b) as being approximately 4 cm. We assume a joint

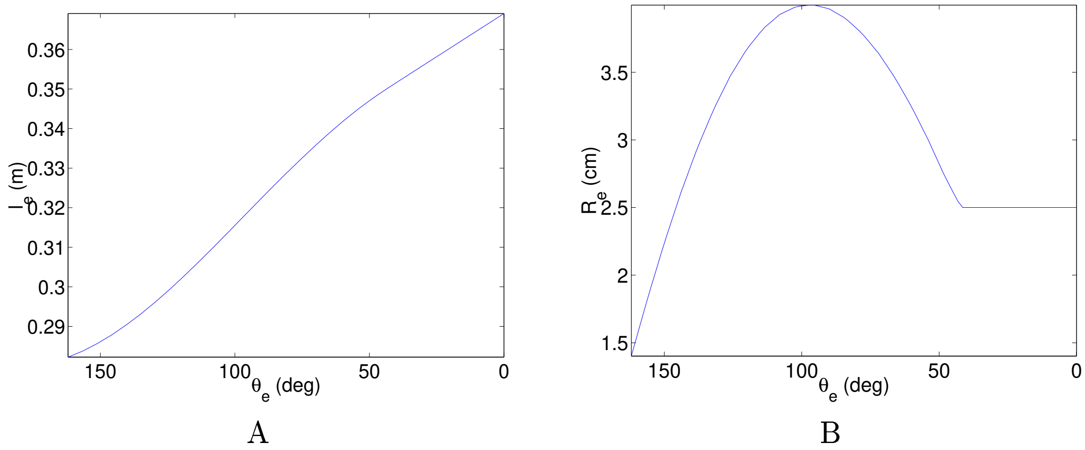

Figure 3: Mono-articulate muscle length (A) and moment arm (B) as a function of joint angle.

capsule radius of 2.5 cm (which is consistent with that measured by Amis et al. 1979). Although the origin of the biceps is located on the scapula, the path constraints imposed by the surrounding tissue are such that the *effective origin* is located near the tip of the intertubercular groove (Högfors et al., 1987). We thus take LM_a to be 32 cm. [2] These parameters have been also used by Gribble et al. (1998), and are similar to those used by van Zuylen et al. (1988) in their model of moment arm variation of the biceps ($LA_a = 31\ cm$; $LA_b = 4.5\ cm$). Furthermore, An et al. (1981) report similar parameters in their study of human moment arm variation ($r \approx 1.5\ cm$; $LA_b \approx 3.5\ cm$). The muscle length and moment arm variations for this set of parameters are shown in figure 3.

For the shoulder, less is known about the effective origin and insertion for the muscle. We assume for the purposes of our model that $LA_a = 5\ cm$, $LA_b = 5.1\ cm$, and $r = 3.5\ cm$.

3 The Biarticulate Case

The biarticulate case, in which the muscle can wrap around both the shoulder and elbow joint capsules is a generalization of the single-joint case. However, we now have four separate cases to consider: whether or not the muscle wraps around each joint.

3.1 Parameter/Variable Definitions

Figure 4 shows the basic structure of the model. Given parameters: L_1 (length of the upper-arm segment), r_1 and r_2 (the radii of the shoulder and elbow joint capsules, respec-

[2]The distance between the centers of rotation of the shoulder and elbow are taken to be 34 cm

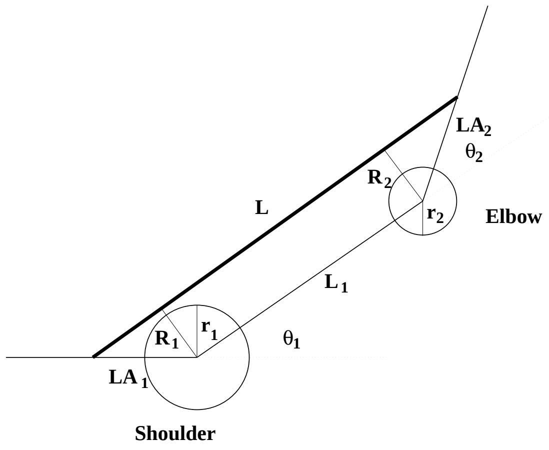

Figure 4: Model Structure

tively), and LA_1 and LA_2 (the distance from the point of rotation to the muscle attachment point); and the variables θ_1 and θ_2 (current orientation of the shoulder and elbow, respectively), we must compute: R_1 and R_2 (the moment arms about the shoulder and elbow), and L (the length of the muscle).

3.2 Joint Space Partition

The joint space is partitioned into four regions, as shown schematically in figure 5. The location of the boundaries depends upon the arm and muscle attachment parameters.

3.2.1 Double-Wrap Boundary

The most extreme point of the double-wrap region occurs when the muscle glances both joint capsules (figure 6). In other words, contact with each capsule is a single point, with no wrapping of the muscle. The associated quantities are computed as follows:

$$\alpha_1 = \arccos\left(\frac{r_1}{LA_1}\right), \tag{6}$$

$$\alpha_2 = \arccos\left(\frac{r_2}{LA_2}\right), \tag{7}$$

$$\gamma = \arccos\left(\frac{r_1 - r_2}{L_1}\right), \tag{8}$$

$$\theta_1^c = \pi - \alpha_1 - \gamma, \tag{9}$$

$$\theta_2^c = \gamma - \alpha_2, \tag{10}$$

$$LM_1 = LA_1 \sin\left(\alpha_1\right), \tag{11}$$

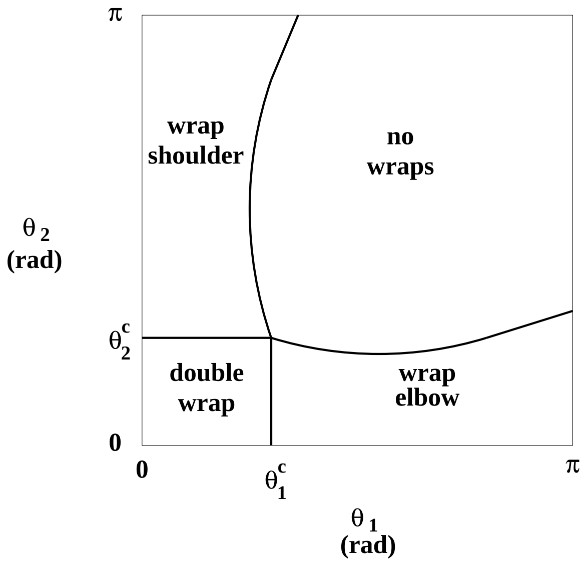

Figure 5: Partitioning of the joint space as a function of whether the muscle wraps around the two joint capsules.

$$LM_2 = LA_2 \sin(\alpha_2), \tag{12}$$
$$LM_w = L_1 \sin(\gamma), \tag{13}$$
$$L = LM_1 + LM_w + LM_2,$$

where θ_1^c and θ_2^c are the joint angles at which this condition occurs. As long as $\theta_1 \leq \theta_1^c$ and $\theta_2 \leq \theta_2^c$, the muscle wraps around both capsules. LM_1 and LM_2 represent the distance from the muscle's point of attachment to the corresponding joint capsule contact point; LM_w is the distance between the two contact points.

The double-wrap condition exists if $\theta_1 < \theta_1^c$ *and* $\theta_2 < \theta_2^c$.

3.2.2 Shoulder Wrap Boundary

As long as the double-wrap condition does not occur, and $\theta_1 - \theta_1^c < \theta_2 - \theta_2^c$, then the possibility exists that the muscle will wrap around the shoulder joint capsule. However, the boundary varies as a function of θ_2. Therefore, the strategy that we take is to assume no wrapping, compute the shoulder moment arm, and then compare that moment arm to the radius of the joint capsule. If $R_1 < r_1$, then we have the case in which the muscle is wrapping around the shoulder joint capsule. These moment arm computations for the no-wrap condition are illustrated in figure 7. The process is illustrated below:

$$x_a = L_1 \cos(\theta_1) + LA_2 \cos(\theta_1 + \theta_2), \tag{14}$$

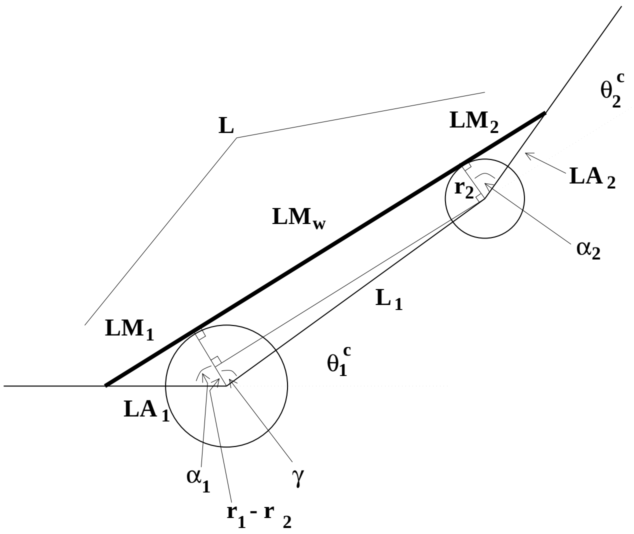

Figure 6: Defining the double-wrap boundary.

$$
\begin{aligned}
y_a &= L_1 \sin(\theta_1) + LA_2 \sin(\theta_1 + \theta_2), && (15)\\
\omega_1 &= atan2\,(y_a, x_a + LA_1) \\
R_1 &= LA_1 \sin(\omega_1), && (16)\\
\omega_2 &= \frac{\pi}{2} - \theta_1 - \theta_2 + \omega_1, \\
R_2 &= LA_2 \cos(\omega_2), && (17)
\end{aligned}
$$

where $< x_a, y_a >$ denotes the location of the muscle attachment to link 2 relative to the center of rotation of the shoulder, and R_1 and R_2 are the moment arms.

Returning to the question of whether the muscle wraps around the shoulder joint capsule – this is the case as long as $R_1 \leq r_1$ **and** $\theta_1 < \frac{\pi}{2}$. The latter condition is necessary because when the shoulder is flexed to a large degree, the moment arm drops below the r_1 threshold, and yet the geometry is such that the muscle does not wrap around the joint capsule.

3.2.3 Elbow Wrap Boundary

Determining the boundary for the elbow wrap case follows in the same manner as the shoulder. Given that the double-wrap condition does not apply, the elbow-wrap case applies if $\theta_1 - \theta_1^c < \theta_2 - \theta_2^c$, $R_2 \leq r_2$, and $\theta_2 < \frac{\pi}{2}$ (where R_2 is the same as in equation 17).

3.3 Computing Muscle State

Now that we have determined the wrapping state for a given arm configuration ($< \theta_1, \theta_2 >$), we can now compute the muscle moment arms (R_1 and R_2) and muscle length

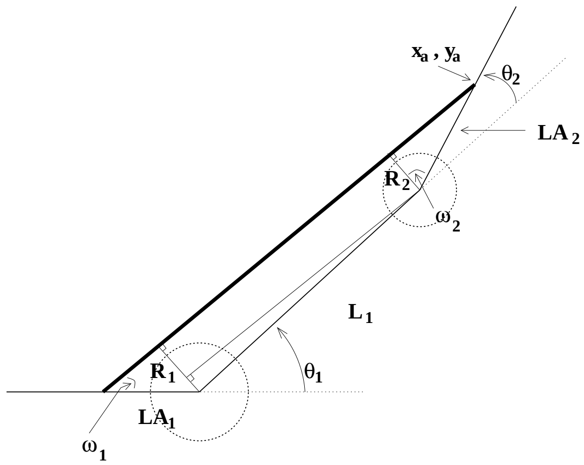

Figure 7: No joint capsule wrapping.

(L).

3.3.1 No Wrap Case

Refering to figure 7, the muscle length, L is simply:

$$L = \sqrt{(x_a + LA_1)^2 + y_a^2},$$

where x_a and y_a are from equations 14 and 15. The muscle moment arms are exactly what have been computed in equations 16 and 17.

3.3.2 Shoulder Wrap Case

The shoulder wrap case is illustrated in figure 8. The corresponding quantities are computed as follows:

$$\begin{aligned}
b &= \sqrt{L_1^2 + LA_2^2 + 2L_1LA_2\cos(\theta_2)}, \\
\eta &= \arccos\left(\frac{r_1}{b}\right), \\
\varphi &= \arccos\left(\frac{L_1 + LA_2\cos(\theta_2)}{b}\right), \\
\beta_1 &= \pi - \eta - \varphi - \alpha_1 - \theta_1, \\
\lambda &= \frac{\pi}{2} - \alpha_1 - \beta_1 - \theta_1,
\end{aligned}$$

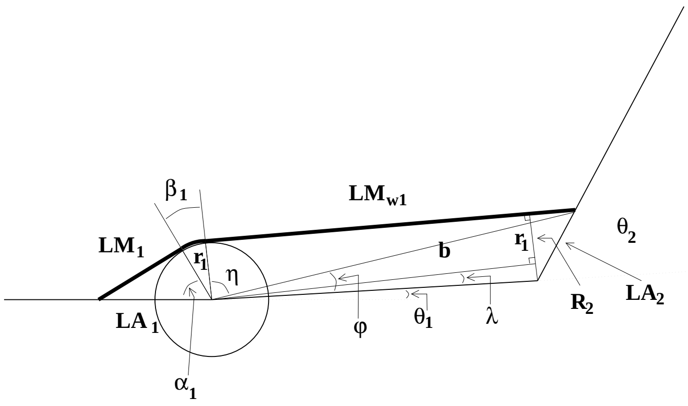

Figure 8: Wrap around shoulder joint capsule.

$$
\begin{aligned}
L &= LM_1 + r_1\beta_1 + b\sin(\eta), \\
R_1 &= r_1, \\
R_2 &= r_1 + L_1\sin(\lambda),
\end{aligned}
\tag{18}
$$

where α_1 and LM_1 are from equations 6 and 11, respectively.

3.3.3 Elbow Wrap Case

The elbow wrap case is illustrated in figure 9. The associated quantities are computed as follows:

$$
\begin{aligned}
\hat{b} &= \sqrt{L_1^2 + LA_1^2 + 2L_1LA_1\cos(\theta_1)}, \\
\xi &= \arcsin\left(\frac{r_2}{\hat{b}}\right), \\
\rho &= \arccos\left(\frac{LA_1 + L_1\cos(\theta_1)}{\hat{b}}\right), \\
\beta_2 &= \frac{\pi}{2} - \theta_1 - \theta_2 - \alpha_2 + \rho + \xi, \\
L &= \sqrt{\hat{b}^2 - r_2^2} + \beta_2 r_2 + LM_2, \\
R_1 &= LA_1\sin(\rho + \xi), \\
R_2 &= r_2,
\end{aligned}
\tag{19}
$$

where α_2 and LM_2 are from equations 7 and 12, respectively.

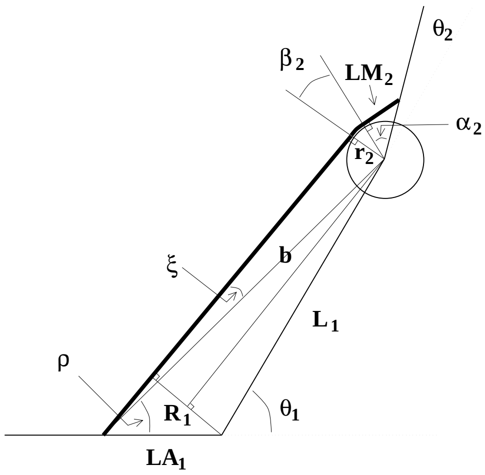

Figure 9: Wrap around elbow joint capsule.

3.3.4 Double Wrap Case

The double wrap case is illustrated in figure 10. The relevant quantities are computed as follows:

$$\begin{aligned}
\beta_1 &= \pi - \alpha_1 - \theta_1 - \gamma \\
\beta_2 &= \gamma - \alpha_2 - \theta_2 \\
L &= LM_1 + \beta_1 r_1 + LM_w + \beta_2 r_2 + LM_2 \\
R_1 &= r_1 \\
R_2 &= r_2
\end{aligned} \tag{20}$$

where α_1, α_2, γ, LM_1, LM_2, and LM_w are as computed in equations 6-13.

3.4 An Example

We use the following parameters for the biarticulate flexor:

$$\begin{aligned}
LA_1 &= 5\ cm, \\
LA_2 &= 4\ cm, \\
r_1 &= 3.5\ cm, \\
r_2 &= 2.5\ cm, \\
L_1 &= 34\ cm.
\end{aligned}$$

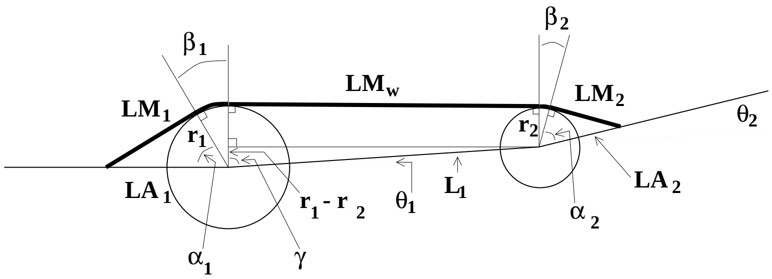

Figure 10: Wrapping around both joint capsules.

Figure 11 shows the arm and muscle in a no-wrap configuration, and the joint space boundaries defined by the muscle wrapping state. Variations of the muscle length and joint moment arms are shown in figures 12 and 13. Note that for both joints, at the extreme limits ($\theta_S = \pi$ for the left panel, and $\theta_E = \pi$ for the right panel), the moment arms become negative. In these cases, the muscle applies torques in the direction that is opposite from normal (for the biceps short head, this situation cannot occur physically due to joint limit constraints, however it is known to happen for other arm muscles).

3.5 Practical Issues

In practice, we have found that it is more efficient to use the equations presented here to construct a lookup table for muscle length and moment arm as a function of joint angle(s) rather than using the equations directly during a dynamic simulation. We then perform interpolation using elements of the lookup table.

4 Extensor Parameters

We assume that the extensors maintain a constant moment arm. The monoarticulate shoulder and elbow muscles, we assume moment arms of 3.5 cm and 2.5 cm, respectively. For the biarticulate muscle, we assume a moment arm of 4.0 cm and 2.5 cm for the shoulder and elbow, respectively. These values are consistent with those used by Gribble et al. (1998).

References

Amis, A. A., Dowson, D., and Wright, V. (1979). Muscle strengths and musculo-skeletal geometry of the upper limb. *Engineering in Medicine*, 8(1):41–48.

An, K. N., Hui, F. C., Morrey, B. F., Linscheid, R. L., and Chao, E. Y. (1981). Muscles across the elbow joint: A biomechanical analysis. *Journal of Biomechanics*, 14(10):659–669.

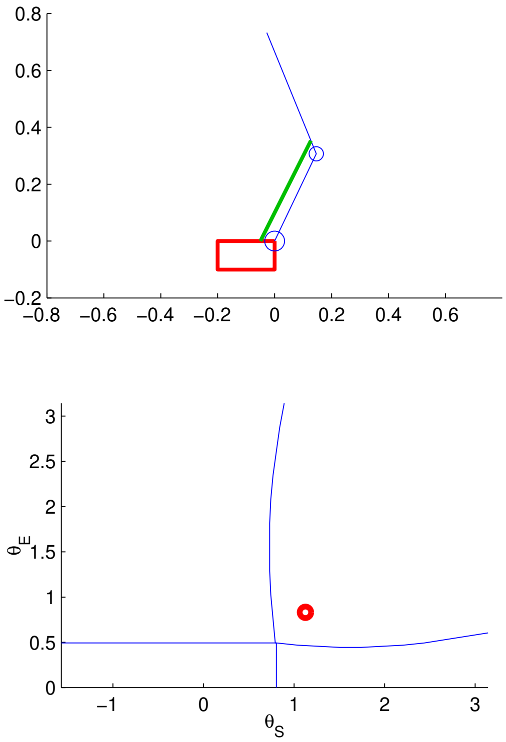

Figure 11: An example arm configuration - Cartesian- (upper) an joint-space (lower). The joint-space plot shows the actual space partitioning as a function of muscle wrapping around the joint capsules. Note that in this plot, θ_E corresponds to θ_1 in the derivation, and θ_S corresponds to θ_2.

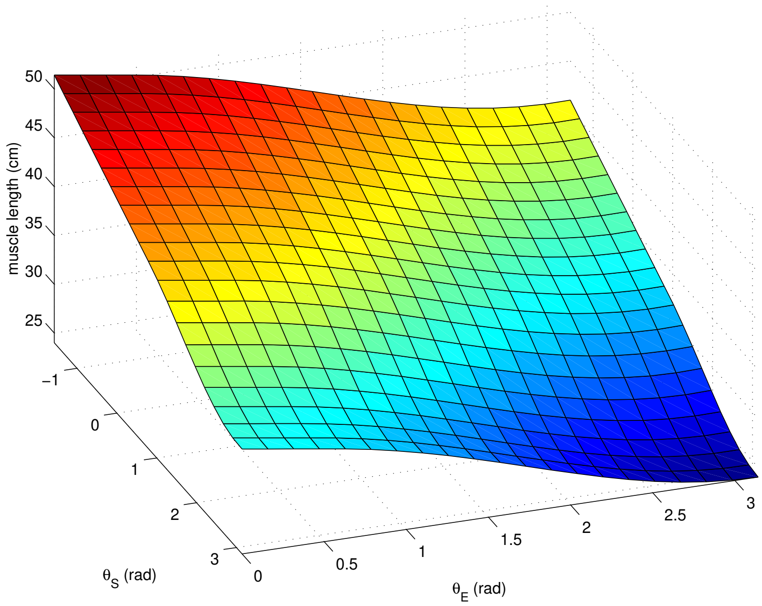

Figure 12: Biarticular muscle length as a function of joint angle.

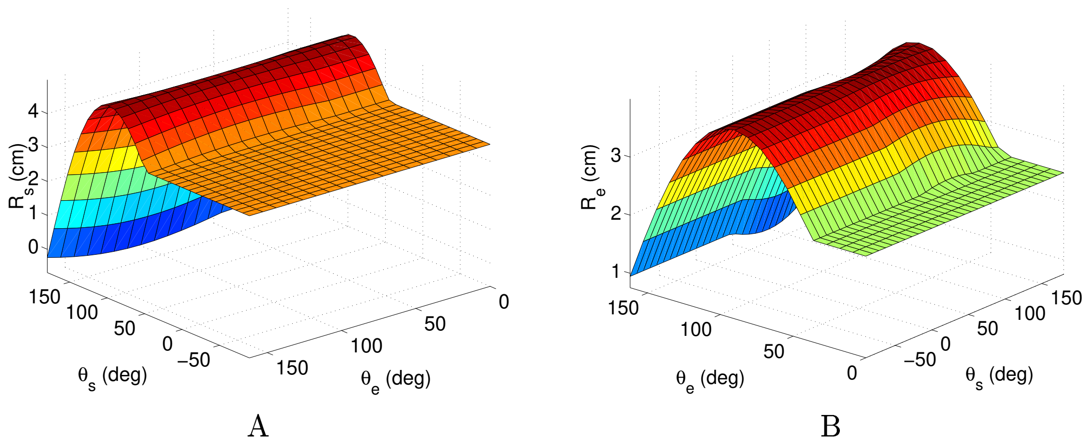

Figure 13: Biarticular moment arm as a function of joint angle for the shoulder (A) and elbow (B).

Fagg, A. H., Houk, J. C., and Barto, A. G. (2001a). Fractional power damping model of control I. muscle/spinal dynamics. *in preparation*.

Fagg, A. H., Houk, J. C., and Barto, A. G. (2001b). Fractional power damping model of control II. joint control. *in preparation*.

Gribble, P. L. (1998). personal communication.

Gribble, P. L., Laboissière, R., and Ostry, D. J. (1997). Control of human arm and jaw motion: Issues related to musculo-skeletal geometry. In Morasso, P. and Sanguineti, V., editors, *Self-Organization, Computational Maps, and Motor Control*, pages 483–506. Elsevier Science.

Gribble, P. L., Ostry, D. J., Sanguineti, V., and Laboissière, R. (1998). Are arm control signals required for human arm movement? *Journal of Neurophysiology*, 79(3):1409–1424.

Högfors, C., Sigholm, G., and Herberts, P. (1987). Biomechanical model of the human shoulder i. elements. *Journal of Biomechanics*, 20(2):157–166.

van Zuyeln, E. J., van Velzen, A., and Denier van der Gon, J. J. (1988). A biomechanical model for flexion torques of human arm muscles as a function of elbow angle. *Journal of Biomechanics*, 21(3):183–190.

Winters, J. M. and Stark, L. (1988). Estimated mechanical properties of synergistic muscles involved in movements of a variety of human joints. *Journal of Biomechanics*, 21:1027–1041.