

Nonlinear Damping Dynamics and the Variability of Rapid Aimed Movements

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Abstract

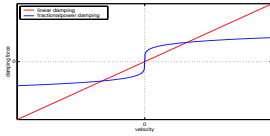
We describe a set of computational experiments aimed at studying the hypothesis that key properties of variability in fast reaching movements are due to nonlinear dynamical properties of the plant. Specifically, we studied ballistic single degree-of-freedom movements generated by a fractional-power damping model of the plant driven by various pulse-step motor commands. We show that fractional-power damping naturally produces both linear and logarithmic relationships between movement velocity and end point variance.

Fractional-power damping model

The fractional-power damping model arises from certain simplifying assumptions about muscle mechanical properties and spinal reflex mechanisms (Houk 1981; Wu et al. 1990). The simplifying assumptions reduce the muscle stiffness of the muscle-reflex system to a linear stiffness. Fractional-power damping in the model arises from the interaction of muscle properties and the friction-like property of the stretch reflex. The simplest model that captures the above mentioned critical features is a spring-mass system with fractional-power damping term:

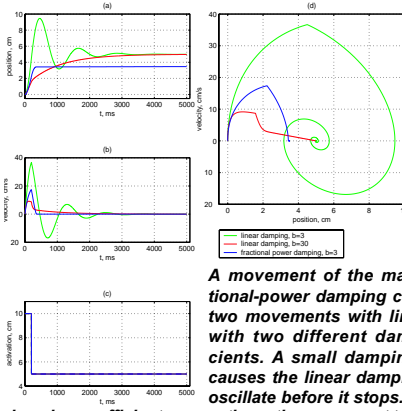
$$m \ddot{x} + b \dot{x}^{\frac{1}{5}} + k(x - u) = 0.$$

To account for dynamic properties of muscle cell activation and of signal transfer within the central nervous system, the dynamic model was augmented with the second-order low-pass temporal filter.



Fractional-power damping causes a relatively high damping force for lower velocities and relatively low damping force for higher velocities.

The low damping for higher velocities allows the mass to move fast from one location to another. The high damping for lower velocities allows it to stop efficiently without waiting too long for oscillations to decay. These features make it possible to accomplish movements faster and to stop in a desired location relatively easily.



A movement of the mass with fractional-power damping compared with two movements with linear damping with two different damping coefficients. A small damping coefficient causes the linear damping system to oscillate before it stops. A large

damping coefficient causes the entire movement to be very slow. Fractional-power damping allows the combination of fast movements with efficient stopping without oscillations.

Fractional-power damping gives rise to dynamic behavior that includes a "stiction region": an extended region in the state space where movement effectively stops away from the system's equilibrium state (Barto et al., 1999). For each fixed control signal, the stiction region has well-defined borders, and final position variability can only take place within the region.

Experiments

The purpose of the experiments was to assess the final position variability for a series of similar movements with noisy control signals or plant dynamics. To produce different movements we used pulse-step combinations with different pulse magnitudes with the same pulse duration and the same step magnitude. For each particular pulse magnitude, we simulated a large number of movements while various parameters of the control signal or the state of the plant were perturbed. We perturbed a chosen parameter one at a time while all other parameters remained fixed for the particular set of movements. We have studied several noise injection models: **stochastic pulse magnitude**, **stochastic pulse duration**, **stochastic plant dynamics**. In all these noise injection schemes the variance of the final position arises due to the presence of the stiction region. The main purpose of this research was to investigate how the final position variance depends on the pulse magnitude and average velocity, amplitude, and duration of the movements.

Results

For a set of movements generated by noisy command signals, the speed-accuracy relationship critically depends on the proportion of movements that hit the inner area of the stiction region versus the proportion of movements that undershoot or overshoot the region, thereby effectively stopping on an edge of the region. These results provide a new perspective on possible mechanisms for both the linear and logarithmic speed-accuracy relationships observed in reaching.

The graphic panels at the top of the right column show representative movements generated in our experiments and the speed-accuracy relationship data with the corresponding linear and logarithmic fits. Only data for stochastic pulse duration are shown. Results obtained for stochastic pulse magnitude and stochastic plant dynamics are similar.

Discussion

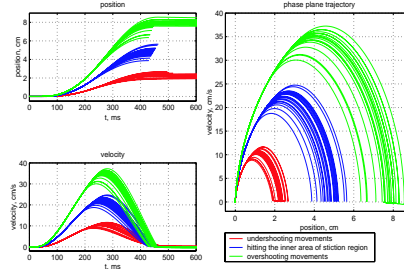
The obtained data stand in good agreement with the hypothesis on linear speed-accuracy relationship (e.g. Schmidt et al., 1979; Wright and Meyer, 1983):

$$\sigma = c + d \cdot \frac{A}{MT}.$$

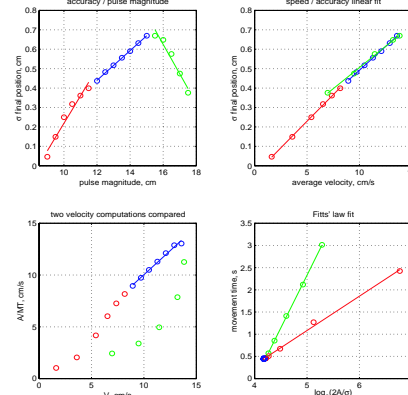
The data for undershooting sets and for overshooting sets are well fit by Fitts' law (Fitts, 1954) as well:

$$MT = a + b \cdot \log_2 \left(\frac{2A}{\sigma} \right),$$

here σ is the standard deviation of final positions for a set of movements, A is the average amplitude for a set of movements, MT is the average movement time, a , b , c , and d are the corresponding fit parameters.



Representative movements from sets of three different kinds: a set containing movements **undershooting the stiction region, a set with all movements hitting the **inner area** of the stiction region, a set containing movements **overshooting** the stiction region.**



Each point on each graph corresponds to one set of movements like those shown above. Data for sets containing **undershooting movements, sets with all movements hitting the **inner area** of the stiction region, and sets containing **overshooting** movements are well fit by three different linear functions. Data for the **undershooting** and **overshooting** sets are well fit by Fitts' law. The lower-left panel shows the results of two different methods for computing average velocities (see below). The two methods produce identical results for **inner area** sets, and different results for **undershooting** and **overshooting** sets.**

This is an obvious contradiction, especially if we notice that the intercept c in our data is equal to zero:

$$\sigma = c + d \cdot \frac{A}{MT}, \quad c \approx 0 \implies MT = d \cdot \frac{A}{\sigma},$$

A possible explanation is that in the linear relationship formula the average velocity V is used instead of A/MT :

$$V = \frac{1}{n} \sum \frac{A_i}{MT_i},$$

while Fitts' law formulation always averages A and MT separately:

$$A = \frac{1}{n} \sum A_i, \quad MT = \frac{1}{n} \sum MT_i,$$

here A and MT are respectively the amplitudes and the movement times of separate movements, n is the number of movements in the set (n was equal 10,000 for all sets). In the general case these two computations produce different results:

$$\exists \{A_i\}, \{MT_i\}: \frac{\sum A_i}{\sum MT_i} \neq \frac{1}{n} \sum \frac{A_i}{MT_i}.$$

In our data the two computations produce identical results for sets of movements which hit the inner area of the stiction region, while for sets containing undershooting or overshooting movements the results are completely different. This is shown in the lower-left panel above.

References

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